# An Approach to Robust Decision Making under Severe Uncertainty in Life-Cycle Design

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**Abstract**

Information-Gap Decision Theory (IGDT), an approach to robust decision making under severe uncertainty, is newly considered in the context of a simple life cycle engineering example. IGDT offers a path to a decision in the class of problems where a nominal estimate of an uncertain life cycle parameter is available, but the amount of the deviation of that estimate from the actual value, as well as the implications of that deviation on performance, are not known. The decision rule inherent in IGDT entails relaxing one's demand for optimal performance and choosing designs with maximum immunity, or *info-gap robustness*, to the effects of deviation from the known estimate. This tradeoff is analyzed graphically using plots of robustness versus performance demand. In this paper, an automotive oil filter design example affected by severe uncertainty is formulated and solved using an IDGT approach. The types of life cycle engineering design problems that the approach could be effective towards are discussed, as are potential limitations that could be encountered when solving more complex problems.

**Keywords**

Information-Gap Decision Theory, Uncertainty, Decision Making, Robustness, Life Cycle Design.

## 1 INTRODUCTION

Uncertainty is prevalent in complex systems, especially regarding time-distant aspects, and it often complicates engineering design choices. Recent practitioners have considered uncertainty due to lack of information separately from random variability, which is irreduceable and can be represented probabilistically [1]. Uncertainty due to deficient information can be encountered in any of the components that comprise a multicriteria assessment, as shown partitioned in Figure 1. Components in the figure are grouped, as indicated by dashed-lines, using Hofstetter's concept of technical, ecological, and value "spheres" of knowledge and reasoning about environmental performance evaluation [2].

Considering only the "Life Cycle Events" component of Figure 1, uncertainty can come from numerous sources and can be difficult to reduce for various reasons. Uncertainty is most severe where customer population, distribution, and behavior is unknown, or where shifting regulations, resource availability, or consumer priorities affect the waste processing, material-cycling or energy-supply infrastructures. Inability to collect information can be due to the sheer complexity and expense of modeling these systems, as well as due to limited historical data about the products themselves, or limited information-gathering resources because of shortened or concurrent product development timelines.

This paper will investigate severe uncertainty in life cycle *events*. The decision scenario examined herein entails selection between concepts, where a decision maker's preference for one or the other could switch depending on the actual value an uncertain quantity takes. The form of severe uncertainty considered is particularly confounding because of unknown bounds on imprecision, i.e., unknown size of deviation between a nominal estimate and the actual unknown quantity. To confront this, information-gap decision theory (IGDT) will be used to determine whether a "good enough" design alternative could be more robust to severe lack of information about uncertainty bounds [3]. Rather than trying to add to info-gap theory, the purpose of this paper is to walk through a design example and explain the implications of the theory in a life cycle design context.

## 2 DECISIONS UNDER LIMITED INFORMATION IN LIFE CYCLE DESIGN

There are a variety of design techniques that factor the effects of uncertainty into decision making, including use of subjective probabilities [4], intervals [5], possibility theory [6], imprecise probabilities [7], and evidence theory [8]. Approaches to dealing with uncertainty in environmental assessments, as proposed by the life cycle assessment (LCA) community, appear limited to using one or more of the first three of these techniques [9-11]. Some also use a taxonomy to decide which apply to the different forms of uncertainty in the life cycle [12, 13]. All of the preceding techniques and combined approaches rely on either multiple data samples or subjectively defined distributions or intervals based on expert belief.

There are circumstances, however, where there is nothing available to describe an uncertain variable other than a

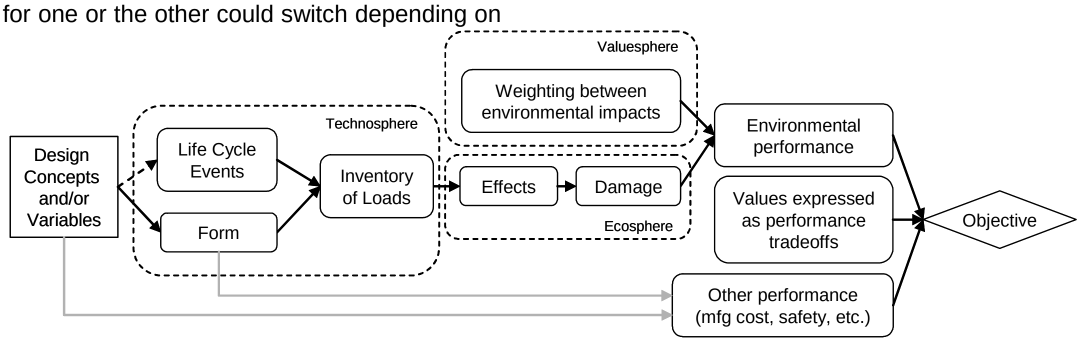


Figure 1: Structure of a multi-criteria product life cycle assessment.

nominal estimate (e.g., an approximation, a comparable baseline, expert estimate, etc.), with upper and lower error bounds on either side of that estimate unknown. Common responses to such severely deficient information include postponing decision making entirely, haphazardly collecting more data without a clear understanding of its value, or even relying on unwarranted assumptions that fill in missing information.

*Information-gap decision theory* (IGDT), developed by Ben-Haim [3], is an alternative approach to making design decisions when there is an unknown gap between an uncertain quantity's available (but suspect) nominal value and its true value, the latter of which *could* be known but is not. IGDT models the size of the gap between the known and unknown as a free *uncertainty parameter*, $\alpha$. To confront this gap, the design decision maker must specify a *satisficing* performance level—a "good enough" minimum level acceptable in a worst case scenario—and accordingly choose the design that, subject to that survival requirement, safely allows for the greatest amount of error, i.e., the largest $\alpha$. This choice is based on a satisficing, robustness-maximizing decision rule, which can be preferable to a performance-optimizing rule applied amidst deficient information. The fact that the uncertainty parameter $\alpha$ is initially unspecified, with robustness to its unknown size *maximized* in the search across the design space, makes IGDT different than other decision approaches.

IGDT has steadily evolved over 15 years from a body of work on convex set-based models of uncertainty [14-16] and has been used in a variety of applications, including management of natural systems flood management [17], water resources management [18], correlation studies between experimental tests and simulations [19], structural design [20, 21], and biological conservation management [22]. A recent application of IGDT to pressure vessel design has explored implications for design problems with continuous design variables [23].

## 3 IGDT CONCEPTS AND COMPONENTS

Instead of optimizing performance, IGDT optimizes a *robustness function* subject to a satisficing constraint on performance or reward. Satisficing means accepting designs with "good enough" performance in order to afford the potential to achieve other objectives, especially when only idealized models or limited information is available [24]. Using IGDT, one can determine whether, under a satisficed demand for performance, a design can be found that affords more immunity to the effects of approximation error of unknown size. Towards this end, the robustness function, $\hat{\alpha}(q,r_c)$, quantifies the maximum uncertainty level that can be sustained while still guaranteeing that a desired critical level of performance is met. The "robust-optimal" design, $\hat{q}$, is the one that can endure the largest uncertainty size. This design often differs from the performance-optimal choice normally sought in optimization.

The foundational theory presented subsequently in this section can be found in [3]. The three components needed for an info-gap analysis are:

1. A performance (or "reward") model, $R(q,u)$, of system response that is a function of an uncertain variable, $u$, and (continuous or discrete) design option(s), $q$; and whose output is a performance attribute of interest.
2. $u$, the uncertain variable, representable as an info-gap and relating to (1) above. Actually, $u$ can also be a model or function, not considered here.
3. $r_c$, a critical satisficing value of performance that must be guaranteed; alternatively considered a failure criterion.

In IGDT, one only knows that there is uncertainty associated with a particular quantity, and knows an estimate of the nominal value for that quantity, but does not know the size of the uncertainty for that quantity. As shown in Figure 2, uncertainty, $u$, is represented as nested, convex sets centered around a nominal value, $\tilde{u}$. The size of each set is characterized by the free uncertainty parameter, $\alpha$. Mathematically, a simple uniformly bounded info-gap can be defined as:

$$u = \mathrm{U}(\alpha,\tilde{u}) = \left\{u : \left|u-\tilde{u}\right| \le \alpha\right\}, \alpha \ge 0 \tag{1}$$

The info-gap model, parameterized from its center, has two ends of interest for each set in the family, as seen in Figure 2. The focus of this paper will only be on the bound that creates the worst consequence to performance. However, IGDT can consider the "better" end of the interval when using an *opportunity function* [3], not discussed herein.

Info-gap models are defined based on information about how the bounds on the uncertain variable grow. Besides the uniform bound model of Eq (1) and Figure 2, info-gaps can be bounded using various envelope types as discussed in [3]. If $u$ is itself a function or model, then integral, Fourier, or other types of bounds can be defined.

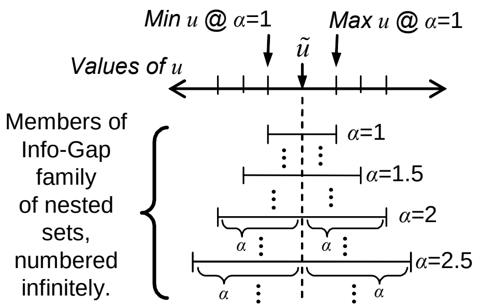


Figure 2: Representing unbounded uncertainty as an *α*–parameterized family of nested sets, adapted from [23].

From the three IGDT components, a robustness function can be defined that maximizes the size that the uncertainty parameter $\alpha$ can take as still satisfy the satisficing constraint. When increased $R(q,u)$ is desirable, the satisficing constraint is:

$$R(q,u) \ge r_c \tag{2}$$

This constraint is embedded into the robustness function, defined mathematically as an optimization problem:

$$\hat{\alpha}(q,r_c) = \max\left\{\alpha : \min_{u\in U(\alpha,\tilde{u})} R(q,u) \ge r_c\right\} \tag{3}$$

To help interpret this expression, the info-gap robustness is "the maximum tolerable $\alpha$ so that all $u$ [in the info-gap model's family of sets] up to uncertainty size $\alpha$ satisfy the minimum requirement for survival" [3]. Eq. (3) is formulated for cases where larger values of performance are better. If, on the other hand, *smaller* performance is desired, as when the objective involves stress, cost, or environmental impact, a maximization should replace the inner minimization:

$$\hat{\alpha}(q,r_c) = \max\left\{\alpha : \max_{u\in U(\alpha,\tilde{u})} R(q,u) \le r_c\right\} \tag{4}$$

In either case, the "hat" on the symbol for robustness, $\hat{\alpha}$, distinguishes it from uncertainty size $\alpha$. The actual value of $\alpha$ is unknown, but one can still determine how much robustness, $\hat{\alpha}$, to deviation between the known nominal and unknown actual can be gotten by choosing a satisficing design rather than a risky performance-optimal one. If the satisficing constraint, $r_c$, is flexible, one can examine the effect that relaxing the requirement has on opportunity for info-gap robustness, and develop a preference for a tradeoff. Such a tradeoff will be examined in the subsequent example.

An IGDT problem can be formulated with continuous or discrete design variables, one or many performance dimension $R$, and even multiple uncertainties $u$. However, for an elementary introduction to applications in life cycle design, it will next be applied to a simple binary selection problem with one uncertainty.

## 4 EXAMPLE APPLIED TO LIFE CYCLE DESIGN

A manufacturer of filters is looking to expand its product line with a new oil filter design for automobile engines. Engineers have developed two competing concepts for the new design: a "take-apart spin-on" (TASO) variant with a reusable aluminum casing and disposable filter, and a "steel easy change" (SEC) variant that is completely disposable. It happens that the functional performance and cost of the alternatives are equal, so the factor determining design preference is their environmental burden over the life cycle. Since the TASO has a reusable casing that lasts the lifetime of an engine, the environmental performance of both alternatives is considered over the total number of filters used, *F*, throughout one engine lifetime.

Which design has the greater burden depends on the number of lifetime filter changes. The dilemma faced is that, despite the fact that a mileage period between filter changes will be recommended, uncertainty about the frequency with which the customers will actually change their filters, coupled with uncertainty about the average lifetime of their cars, makes the actual average number of lifetime filter changes severely uncertain. The designers wish to evaluate the selection decision without collecting further information, and decide to use the IGDT approach to do so.

To walk through an example, an oil filter design selection problem will first be defined generically and then formulated and solved as an info-gap decision analysis problem. The formal IGDT notation and symbolic solution is presented first; however, the implications of satisficing are most clearly seen through the graphical presentation and discussion that comes afterwards.

### 4.1 Design Problem Scenario

Both filter designs include two main components: a *casing* consisting of a housing plus end caps, and a *cartridge* made of a cellulose filtering material supported by a thin, porous metal inner tube. The cartridge fits inside the casing. The dimensions of all components have been specified for the appropriate balance of strength and weight and are therefore fixed.

For this example, the life cycle environmental burden of each design will be calculated using the Eco-indicator 99 environmental impact scoring system, which uses millipoints (mPt) units [25]. For different combinations of materials and life cycle activities, Eco-indicator 99 takes into account environmental effects, damages, and values and normalizes them per mass, thereby condensing the ecosphere and valuesphere portions of the assessment structure shown in Figure 1. For this problem, e.g.:

$$impact_{steel} = I_{steel} = mass_{steel} \cdot Eco\text{-}indicator_{steel} \qquad (5)$$

While it is intuitive that more waste oil would be drained from the TASO design after a filter change, it is assumed that SEC filters are recycled in a destructive manner which drains their oil content just as much. Thus, only the material content of the filters themselves are used in scoring. Although in practice it is important to communicate fully to decision makers what masses and remaining assumptions were used in determining the Eco-indicator scores, this rest of this step will be skipped for brevity to instead focus on the influence of uncertainty about the number of filters, *F*, used in an engine's lifetime. The functions for environmental impact of the TASO and SEC designs, respectively, are:

$$I_{TASO} = I_{casing} + \left(I_{cartridge} \cdot F\right) = 23.94 + 0.90 \cdot F, \text{ [mPt]} \qquad (6)$$

$$I_{SEC} = \left(I_{casing} + I_{cartridge}\right) \cdot F = (1.26 + 0.90) \cdot F, \text{ [mPt]} \qquad (7)$$

It can be seen that the essential difference between the environmental impact of the designs is their casings: the TASO incurs a higher initial load, whereas the SEC incurs a variable load that increases with increased number of lifetime changes. The TASO casing has a high material burden because it is made of aluminum, which is more resource intensive per unit weight. Moreover, because it is reused throughout the life of the engine, the TASO casing is made thicker to withstand the torque from removal during cartridge changes. In contrast, the cartridge *and* thin steel casing of the SEC filter are discarded, recycled, and replaced with a new filter at every change.

### 4.2 Info-Gap Problem Formulation and Solution

The IGDT approach applies to a problem where information is severely deficient regarding the average number of filters used over an engine's life. The design decision maker takes the attitude that settling for some satisficing level of performance, if guaranteed, is acceptable and preferable to risky optimized performance that relies on unfounded assumptions about unknown bounds. Accordingly, the decision maker seeks the design alternative with maximum robustness to the unknown discrepancy between the unknown *actual* number of filters and a nominal *estimate* based on information from previous designs. The desire to maximize the size to which the discrepancy can safely grow is subject to a satisficing constraint that defines a largest environmental load that can allowably be accepted, one that is "good enough". In this walkthrough of an info-gap analysis, the relationship between demanded performance and info-gap robustness are revealed graphically.

*Info-gap model of the uncertain variable*

The design firm has experience making filters for vehicles owned by customers in the industrial sector who schedule regular maintenance and change filters with predictable frequency. On average, those customers use 17 filters over the life of an engine. However, the design company wishes to expand its business with a new filter design targeting the public sector. Customers in that sector are expected to have less predictable maintenance behavior, and the degree to which their change frequency will deviate with that of industrial customers is unknown.

Thus, the info-gap model for this example can be specified with the knowledge that:

- The nominal value of oil filters used over an engine's lifetime is $\tilde{F}$ =17, taken from information on maintenance rates in the private sector.
- The growth of deviation around nominal can be expressed mathematically as a simple, uniformly-bounded interval.

Combining the uncertainty parameter, $\alpha$, with this sparse information, the info-gap model for lifetime filter usage is:

$$F(\alpha,\tilde{F}) = \left\{F : \left|F-\tilde{F}\right| \le \alpha\right\},\ \alpha \ge 0 \qquad (8)$$

The form of this particular info-gap model can also be expressed more simply:

$$\tilde{F}-\alpha \le F \le \tilde{F}+\alpha \qquad (9)$$

*Reward function and satisficing critical value*

The other two components needed for an info-gap decision analysis are the reward function and satisficing critical value for performance. Seen in Eqs. (6) and (7), a reward function is needed for each alternative design. Expressed in the form of Eq. (2):

$$R(q_1,u) = I(TASO,F) = 23.94 + (0.90\cdot F),\ [\text{mPt}] \qquad (10)$$

$$R(q_2,u) = I(SEC,F) = (1.26+0.90)\cdot F,\ [\text{mPt}] \qquad (11)$$

The designer chooses a critical satisficing value of $I_{critical}$=43mPt, which is the highest level of environmental impact deemed tolerable. Per the form of Eq. (2), the critical satisficing constraint is:

$$I(alt,F) \le I_{critical} \qquad (12)$$

For convenience, the variable *alt* is used to represent the discrete design alternatives, TASO and SEC.

*Info-gap robustness function*

Of main interest in an info-gap analysis is what largest amount of robustness, $\hat{\alpha}(q,r_c)$, to uncertainty is achievable. To reiterate, this robustness is the largest amount of uncertainty $\alpha$ that can be sustained by a design alternative $q$ and still guarantee at least the chosen satisficing performance level $r_c$. Expressed in the form of Eq. (4), the info-gap robustness for this example is:

$$\hat{\alpha}(alt,I_{critical}) = \max\left\{\alpha : \max_{F\in U(\alpha,\tilde{F})} I(alt,F) \le I_{critical}\right\} \qquad (13)$$

For this particular problem, finding the expression for $\hat{\alpha}$ for either design alternative is relatively simple. First, the uncertain variable $F$ in Eqs. (6) and (7) is replaced with $\tilde{F}+\alpha$, the half of the parameterized info-gap model associated with worse performance, e.g.:

$$I(TASO,F) = 23.94 + (0.90\cdot(\tilde{F}+\alpha)) \qquad (14)$$

With this equation form, one can solve for $\alpha$ and calculate $\alpha(alt,I_{critical})$, equivalent in this case to info-gap robustness, $\hat{\alpha}(alt,I_{critical})$. When the reward function, info-gap model, and/or design space $q$ assume more complicated forms, the optimization problem embedded in Eq. (13) can be more difficult to solve.

For the satisficing level $I_{critical}$=43mPt, $\hat{\alpha}(TASO)$ =5.55mPt and $\hat{\alpha}(SEC)$ =2.92mPt. One design is preferable over others when, *amidst the greater amount of deviation* $\alpha$, it can assure performance at or better than the satisficing requirement. So, TASO in this case is "robust-optimal" and preferred over SEC.

## 4.3 Analysis of Preference for a Robustness-Reward Tradeoff

Analysis of preference for the trade off between robustness and acceptable performance is facilitated by examining tradeoff plots. In most info-gap analyses in the literature, this has meant plotting the robustness function, $\hat{\alpha}(q,r_c)$, with $r_c$ as the dependent variable to reveal the robustness-reward curve. In some cases, as with the oil filter example, it is just as easy to develop a preference by plotting the reward, $R(q,u)$, as a function of $\alpha$, i.e., $R(q,u(\alpha))$. This plot is shown in Figure 3, next discussed.

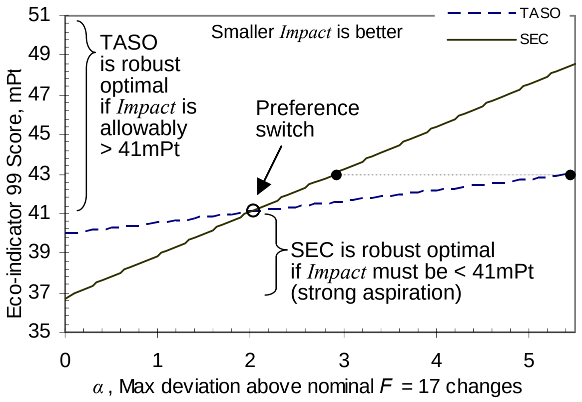


Figure 3: Tradeoff between impact and robustness.

The designer, not knowing the deviation $\alpha$, is tasked with choosing a point on the vertical axis corresponding to his or her demanded level of satisficing performance and, moving rightward perpendicularly from that point, tracing how large uncertainty can grow and still guarantee that performance. For the satisficing level discussed earlier, $I_{critical}$=43mPt, it can be seen that $\hat{\alpha}(TASO)$ =5.55mPt and $\hat{\alpha}(SEC)$ =2.92mPt, as indicated by the dotted horizontal line segment bounded by large dots.

The plot in Figure 3 is instrumental in understanding how design preference changes as the demand for performance is satisficed, or relaxed away from performance-optimal. If an impact as low as 36.7mPt were demanded, only the SEC would satisfy the constraint, and even then, there would be no tolerance for error, $\alpha$, in estimating the number of filter changes. Relaxing the demand to as high as 40mPt, SEC is still the only viable option, with its tolerance for error growing linearly. Above that demand, TASO starts to become a viable option. Though the info-gap robustness it affords is initially less than that of SEC, TASO quickly overtakes, with design preference changing at $I_{critical}$=41mPt, as indicated in Figure 3.

## 4.4 Review of Insight Gained in the IGDT Analysis

The following knowledge is gained in this simple example:

- If the design decision maker can accept a worst-case environmental performance as high (bad) as 41mPt, then the TASO design is preferable because it can endure the highest amount of error above the nominal guess. Moreover, the rate at which info-gap robustness is gained with incremental relaxation of the $I_{critical}$ demand is faster for TASO than SEC, making TASO even more attractive in the presence of extreme uncertainty.
- If there were no uncertainty, SEC would outperform TASO by a difference of 3.22mPt; however, if in reality the deviation above the nominal estimate of 17 filter changes grew as high as 5 changes, for a total

of 22 changes, TASO would then instead outperform SEC by 4.83 mPt.

- Though the designer does not know the level of deviation from nominal, the analysis of the consequences of loss, especially with respect to the point where selection preference changes, gives a point of reference with which to make a decision—effectively, a gamble—if a decision *must* be made under severe uncertainty. If it seems possible that the average number of filter changes could deviate more than 2 above the estimate of 17, the designer should choose the more robust TASO option. It is up to the decision maker to sort out their preference for risk versus guarantee.

While this example is simple and contrived, it serves to introduce the basic concepts and motives for IGDT. It is expected that other uncertainties could be identified in the problem scenario and tested, e.g., the disposal fate of the filters or any of the other assumptions used to total Eco-indicator 99 point values for each design alternative.

## 5 DISCUSSION

Next considered are the pluses and minuses of the IGDT approach, extrapolating learning from the preceding example into questions about wider applications. The need for future work in the area is mentioned throughout.

### 5.1 Evaluation of the Potential of IGDT

In certain situations, the info-gap design analysis approach can eliminate the need for further data collection. For instance, if switching one's design choice (e.g., from SEC to TASO) requires a small sacrifice in guaranteed performance yet affords a reasonably large amount of extra robustness to error in an estimate, one could decide to switch without needing to know more. Future work will attempt to more accurately quantify information cost savings generated by IGDT analyses.

Although info-gap models are meant for use when much less information is available than is required by other existing uncertainty representations, it seems possible that there are still "gray areas" with regards starting information where it difficult to know which approach will produce the best results. Thus, future work will include experiments comparing IGDT results to that of other approaches with different starting information, assumptions, and values, with an aim towards eventually developing a taxonomy for when IGDT would be more appropriate or less expensive to apply.

### 5.2 Limitations and Open Questions

Aspects that could restrict info-gap applications are next explained, including difficulty with value judgements and complications in scaling up to more complex applications.

*Intuitiveness of evaluating severe uncertainty and satisficing reward*

The IGDT approach requires that the decision maker be able to set a critical satisficing performance target and, if needed, weigh and adjust that target in light of the potential for increased info-gap robustness. In the example, we assumed that this could be done for the Eco-indicator 99 measure of environmental impact, as the decision maker chose a satisficing $I_{critical}$=43mPt. However, the Eco-indicator 99 construct was primarily developed to compare options relatively, not absolutely [25]. Still, by definition, 1 Point corresponds to 1/1000 of the environmental load of a European citizen over 1 year [25]. It might be possible to state preference for an absolute score with that reference point in mind; whether this expectation is true is left to future study.

Similarly, a decision maker must be able to relate to the magnitude of the uncertain quantity on which IGDT focuses. While doing so might be feasible for a quantity like lifetime filter changes, the severity of uncertainty in other quantities may be harder to judge. A discussion of calibration and judgment of tradeoffs is considered in [3], but more experimentation is needed to determine the success of decision makers in utilizing these techniques.

*Difficulty of IGDT analysis in complex design scenarios*

Analyzing the relationships between satisficing reward, info-gap robustness, and the robust-optimal design increases in difficulty whenever any of those components are a vector. Having multiple variables for any of them makes visualization and understanding of tradeoffs less intuitive. Existing techniques used to apply IGDT in these situations require assumptions that often are too strict for many applications.

This is especially the case when the influence of two severely uncertain variables are simultaneously analyzed, where two info-gap models with two different $\alpha$ values become necessary. Visualizing a tradeoff of performance to gain robustness to two different info-gaps at the same time may not be intuitive. One technique for multiple uncertainties, applied in a wildlife management application [22], assumed that all $\alpha$ grew at the same rate, which might not be realistic for many problems.

The success with which a decision maker could elicit preferences and choose designs amidst such complexity has not been evaluated, and very little software is available to facilitate the use of IGDT in analysis activities.

## 6 SUMMARY

The information gap decision theory approach has been introduced and used to evaluate the robustness of a decision preference to the unknown error bounds on a nominal estimate. The need for robustness to uncertainty was weighed against relaxing one's demand for guaranteed satisficing performance. This assessment facilitated a decision without added assumptions about bounds or further data collection. A clear demarcation of the effectiveness of info-gap in practical situations, as well as closer examination of the method with respect to other robustness approaches, is left to future work.

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